Appendix 1

State A and B differ only because the *i*th individual has a higher income in state B than in state A. then a social-welfare function is non-decreasing if and only if $W_B \geq W_A$ if $y^{iB} \geq y^{iA}$. A social welfare function is symmetric if

$$W(y^1, y^2, ..., y^n) = W(y^2, y^1, ..., y^n) = ... = W(y^n, ..., y^2, y^1)$$

Social welfare depends on the distribution of income, but not on who gets the income.

A social-welfare function is additive if

$$W(y^1, y^2, ..., y^n) = \sum_{i-1}^{n} U^i(y^i) = U^1(y^1) + U^2(y^2) + ... + U^n(y^n)$$

Appendix 2

$$W = U(y^1) + U(y^2) + ... + U(y^n)$$

Here U is the same for each individual (a consequence of symmetry) and $U(y^i)$ increases with y^i (because the social-welfare function is non-decreasing). If there is an increase in the income of the *i*th individual, the increase in social welfare will be

$$U'(y^i) = \frac{dU(y^i)}{dy^i} > 0$$

A social-welfare function is concave if the welfare weight always decreases as y^i increases – concavity implies diminishing social marginal utility of income. It has constant relative inequality aversion (or constant elasticity) if the utility index U(y^i) has the form

$$U(y^i) = \frac{1}{1-\varepsilon} y^{i(1-\varepsilon)}$$

where ε is a non-negative *inequality aversion* parameter.

Appendix 3

$$U^R = f(Y^R)$$

$$U^P = f(Y^P)$$

where U^R and U^P are the utilities of a representative rich and poor person, respectively, and Y^R and Y^P are their incomes. This is the case implied by an additive social-welfare function and an absolute definition of poverty might be appropriate. But if the utility function are

$U^R = f(Y^R, Y^P), \qquad f_1 > 0, f_2 > 0$

$U^P = f(Y^P, Y^R), \qquad f_1 < 0, f_2 > 0$

Where f_1 and f_2 are the partial derivatives of utility with respect to Y^R and Y^P, respectively, we have an income externality, and both rich and poor might prefer a poverty line that rose over time.

Appendix 4

$P_A = (1 - Y / P)^A$

Appendix 5

Equal opportunity exists if

$E(Y \mid C_i) = K_i$ for all Di

Appendix 6

$W = D / S^E$

Appendix 7

$$V = \frac{1}{n}\sum_{i=1}^{n} (y_i - \mu)^2$$

The disadvantage of variance is its sensitivity to the absolute level of income; if all income doubles, inequality does not change, but V quadruples. To avoid this problem one can use the **coefficient of variation**, defined as

$$C = \frac{V^{0.5}}{\mu}$$

which is the variance normalized on average income. Its advantage is its independence of scale. However, a way of giving greater weight to transfers to lower incomes is to take some transformation that staggers income levels, such as the logarithm. The **variance of the logarithm of income** is this advantage of scale independence.

$$H = \frac{1}{n}\sum_{i=1}^{n} (\log y_i - \log \mu)^2 = \frac{1}{n}\sum_{i=1}^{n} (\log \frac{y_i}{\mu})^2$$

Appendix 8

$$G = \frac{1}{2n^2\mu}\sum_{i=1}^{n}\sum_{j=1}^{n} \left| y^i - y^i \right|$$

$$G = 1 + \frac{1}{n} - \frac{2}{n^2\mu}\left(y^1 + 2y^2 + \ldots + ny^n\right)$$

$$for \quad y^1 \geq y^2 \geq \ldots \geq y^n$$

Appendix 9

$$A = 1 - \left[\sum_{i=1}^{N} \left(\frac{y^i}{\mu} \right)^{1-\varepsilon} f(y^i) \right]^{\frac{1}{1-\varepsilon}}$$

where $\varepsilon \neq 1$