Chapter 3 Basic Notations

Summation

The abbreviated notion for summation is the following: $P_a + P_{a+1} + P_{a+2} \dots \dots P_n = \sum_{k=a}^{n} P_a$ in which $k = a$ indicates the starting number of the sequence, and n indicated the number of repetitions.

An example of an economic application of such a summation is when calculating Price Indices (Inflation) for a group of goods:

$$\frac{Total\ cost\ of\ goods\ in\ final\ year}{Total\ cost\ of\ goods\ in\ Initial\ year} = \frac{\sum_{k=1}^{n} P_F^{(k)} Q^{(k)}}{\sum_{k=1}^{n} P_I^{(k)} Q^{(k)}} \times 100 = \text{Price Index}$$

Where, $Q^{(k)}$ is the amount of good K, $P_I^{(k)}$ is the price of good K in the first year under consideration, and $P_F^{(k)}$ is the price of good K in the final year under consideration.

We can distinguish between two kinds of price indices:

1. Laspeyres Price Index: then the quantity consumed is based on the initial year for the above formula.
2. Paashe Price Index: When the quantity consumed is based on the final year f or the above formula.

Properties of summation

When the variable has a subscript it means that there are different values available for this variable. When there is no subscript, it should be treated as a constant, not varying over time.

1. $\sum_{k=1}^{n}(s_k + r_k) = \sum_{k=1}^{n} s_k + \sum_{k=1}^{n} r_k$ (the additivity property)
2. $r\sum_{k=1}^{n} s_k = \sum_{k=1}^{n} r \times s_k$ (the homogeneity property)
3. $\sum_{k=1}^{n}(s_k + r\ \ ) = \sum_{k=1}^{n} s_k + nr$
4. $\sum_{k=1}^{n} K^2 = \frac{1}{6}n(n+1)(2n+1)$
5. $\sum_{k=1}^{n} K^3 = \left[\sum_{k=1}^{n} K\ \right]^2$
6. $(x+a)^n = \sum_{k=0}^{n}\binom{n}{k}x^k a^{n-k}$, where $\binom{n}{k} = \frac{n(n-1)(n-2)\dots\dots(n-m+1)}{m!}$

   Example: $\binom{7}{3} = \frac{7\times6\times5}{3\times2\times1} = 7 \times 5 = 35$
7. $\sum_{k=1}^{n} s_{k1} + \sum_{k=1}^{n} s_{k2} + \dots + \sum_{k=1}^{n} s_{kb} = \sum_{r=1}^{n}[\sum_{k=1}^{n} s_{kr}]$

   $$\text{Example: } \sum_{r=1}^{2}\left[\sum_{k=1}^{3}(2r+4k)\right] = \sum_{r=1}^{2}[(2r+4) + (2r+8) + (2r+12)]$$

$$= \sum_{r=1}^{2} [6r + 24] = (6 + 24) + \quad (12 + 24) \ = 66$$

Basics of Logic

A proposition, or statement, is an assertion that is either true or false.

1. Implication arrows are used to keep track in a chain of logical reasoning. The implication arrow $\Longrightarrow$ points in the direction of the logical implication. $P \Longrightarrow Q$ means that whenever proposition P is true, proposition Q is necessarily true as well.

2. When the logical implication is true in the other direction as well, we can write $P \Longleftrightarrow Q$, which is a single logical equivalence. This arrow is called an equivalence arrow.

Q is a necessary condition for P ($P \Longrightarrow Q$): a necessary condition for x to be a square (P) is that x be a rectangle (Q). Or as another example, 'the brakes are working (Q)' is a necessary condition for 'the bicycle is in good condition (P)'.

P is a sufficient condition for Q ($P \Longrightarrow Q$): a sufficient condition for x to be a rectangle (Q) is that x be a square (P). A false example would be 'the brakes are working (P)' is a sufficient condition for 'the bicycle is in good condition (Q)', since the brakes could be working while the wheels could be missing. Hence P is not a sufficient condition for Q.

Mathematical proof

This book usually omits formal proofs of theorems, which are the most important results in mathematics. The emphasis lies on an intuitive understanding. Nevertheless:

Every mathematical theorem can be formulated as an implication $P \Longrightarrow Q$, where P represents what we know (premises) and Q represents the conclusions (a series of propositions).

- A direct proof starts from the premises and works towards the conclusions.
- An indirect proof begins by supposing that the conclusion is not true, and reasons that on that basis the premises cannot be true either.

These proofs are both valid, based on the following equivalence:

$P \Longrightarrow Q \quad$ *is equivalent to* $\quad$ *not* $Q \Longrightarrow$ *not* P

Intuitively: 'if it is raining, the grass is getting wet' is the same thing as 'If the grass is not getting wet, then it is not raining'.

These two methods of proof both belong to *deductive reasoning*, reasoning based on consistent rules of logic. Many sciences use *inductive reasoning*, drawing general conclusions on the basis of a few (or many) observations. In mathematics inductive reasoning is not recognized as a form of proof.

Set Theory

In mathematics a collection of objects viewed as a whole is called a *set* and the objects inside the set are called *elements* or *members*. The easiest way to represent a set is to list it between brackets as follows:

$$S = \{a, b, c\}.$$

Two sets are considered to be equal if each element of A is an element of B and vice versa. Then we write $A = B$.

Not every set can be identified by listing all its elements. Some sets contain an infinite number of members. Such sets are common especially in economics. Think for example of the *budget set* in consumer theory. Infinite sets, but also finite sets, can be specified in the following way:

$S = \{typical\ member : defining\ properties\}$

Some standards are convenient to indicate the relationship between a set and its members

- $x \in S$ means that x is an element of S
- $x \notin S$ means that x is not an element of S
- $\emptyset$ is the symbol for an empty set, in other words, a set without elements
- $A \subseteq B$ means that A is a subset of B, and this is true when every member of A is also a member of B. The two sets A and B are only equal when $A \subseteq B$ and

We can distinguish three possible operations with sets:

Notation	Name	This (new) set consists of…	
$A \cup B$	A union B	The elements that belong to at least one of the sets A and B	$A \cup B = \{x: x \in A\ or\ x$
$A \cap B$	A intersection B	The elements that belong to both A and B	$A \cap B = \{x: x \in A\ and\ x$
$A \setminus B$	A minus B	The elements that belong to A, but not to B	$A \setminus B = \{x: x \in A\ or\ x \notin$

If two sets have no elements in common, they are said to be *disjoint*. Sets A and B can only be disjoint when $A \cap B = \emptyset$.

A collection of sets is usually referred to as a family of sets. Each set in any family is a subset of the *universal set* Ω. Hence, the complement of set A, given that set A is a subset of Ω, is the set of element of Ω that are not an element of A. The complement of set A can be denoted by $A^c = \Omega \setminus A$, or $\complement A$. It is always very essential to specify which universal set Ω is used to calculate the complement.

It is often useful to illustrate operations with sets by drawing Venn diagrams. A set is represented by a region in a plane, drawn so that all the elements belonging to a certain set are contained within some closed region of the plane.

These Venn diagrams show that some formulas can easily be found, for example:

- $A \cap B = B \cap A$
- $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

You can find four illustrative Venn diagrams on page 72.

Mathematical induction

Proof by mathematical induction is the following procedure:

Suppose that $A(n)$ is a statement for all natural numbers n and that

a. $A(1)$ is true

b. for each natural number k, if the induction hypothesis $A(k)$ is true, then $A(k+1)$ is true

Then $A(n)$ is true for all natural numbers n.