Chapter 2 Review of basic mathematics, Equations

Solving simple equations
Solving an equation means to find all values of the variables for which the equation is satisfied. If a certain value makes the expression undefined, then that value is not allowed. The trick is to rewrite the equations such that you can find a solution. Hence, first you isolate x, and then you simply solve for x:

Take for example the equation $4x + 5 = 12 - 3x$. The proper procedure to find x is the following:

$$4x - 3x + 5 = 12$$
$$7x + 5 = 12$$
$$7x = 17$$
$$X = 17/7$$

In cases of fractions you will have to find the lowest common denominator in order to solve for x. Take for example the following equation:

$$\frac{4x+5}{x-2} - \frac{9}{x^2-4} = \frac{12}{x+2}$$

We cannot subtract the fractions since the denominators of the fractions are not the same. Therefore we have to rewrite the equation to create the common denominator. In this case the easiest is to choose $x^2 - 4$, since its factor pairs are $(x-2)$ and $(x+2)$. For every fraction we need to multiply the numerator and denominator with the same expression to keep the meaning the same. The algebraic procedure is as follows:

$$\frac{(4x+5)(x+2)}{(x-2)(x+2)} - \frac{9}{(x-2)(x+2)} = \frac{12(x-2)}{(x-2)(x+2)}$$

$$(4x+5)(x+2) - 9 = 12(x-2)$$

$$4x^2 + 13x + 1 = 12x - 4$$
$$4x^2 + x + 5 = 0$$

At this point we would use the quadratic formula to solve for x. However, we will discuss this later on. This example is only meant to illustrate how to get rid of fractions.

Linear equations

The general linear equation is $y = ax + b$, in which y and x are variables, and a and b are called parameters. Parameters can take on different values, but the logic of the equation does not change.

Two examples of linear equations in economics are:

1. $Y = C + \bar{I}$ A country's GDP (Y) equals consumption (C) plus investment ($\bar{I}$) treated as fixed.

2. $C = a + bY$ Consumption (C) is a linear function of GDP (Y)

Together these two are a macroeconomic model. By substituting 2 into 1 we find $Y = a + bY + \bar{I}$ and we can rewrite this into $Y = \frac{a}{1-b} + \frac{1}{1-b}\bar{I}$. This expression gives the *endogenous* variable Y in terms of the exogenous (given) variable $\bar{I}$ and the two parameters. Economists say that the system of two equations is called the structural form and the last equation is called to reduced form, expressing endogenous variables as functions of exogenous variables.

Quadratic Equations

To solve quadratic equations, or second-degree equations we need to find a way to get rid of the powers in the equation. The general quadratic equation is $ax^2 + bx + c = 0, (a \neq 0)$.

Three rules for solving quadratic equations are the following:

1. For this general quadratic equation $ax^2 + bx + c = 0,\ where\ b^2 - 4ab \geq 0\ and\ a \neq 0$

   The solution for x can be found using this formula: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

2. $ax^2 + bx + c = a(x - r)(x - s), if\ ax^2 + bx + c = 0\ and\ r, s\ are\ solutions.$
3. $r + s = -\frac{b}{a}\ and\ r \times s = \frac{c}{a}\ for\ ax^2 + bx + c = 0\ where\ r\ and\ s\ are\ solutions.$

To illustrate these rules, take for example the following quadratic equation:

$$p^2 - 10p + 21 = 0$$

First we illustrate rule 1, the quadratic formula. From the formula we find that a=1, b=-10 and c =21. Using the formula we find two solutions for X:

$$x = \frac{10 \pm \sqrt{100 - 4 \times 1 \times 21}}{2} = \frac{10 \pm \sqrt{16}}{2} = \frac{10 \pm 4}{2}$$
$$= 5 + 2\ or\ 5 - 2, therfore\ 7\ and\ 3\ are\ the\ solutions$$

From rule 3 we deduce that $r + s = -\frac{-10}{1}\ and\ r \times s = \frac{21}{1}$ both have to hold, so r and s should be 7 and 3, as $7 + 3 = 10, 7x3 = 21$. From rule two we can see that the equation can indeed be written as $1(p - 3)(p - 7) = 0$, where 3 and 7 are the solutions.

Linear Equations with two unknowns

An example of a system of two equations with two unknowns is: $4x - 3y = 7$ and $2x + 5y = 23$. We need to solve for both x and y. We review two methods.

1. Substitution Method: First find the value of one variable in terms of the other, then substitute the value into the second equation.

Start by using the first equation to isolate x: $4x - 3y = 7 \Rightarrow 4x = 7 + 3y, x = \frac{7+3y}{4}$

Then substitute the value of x in terms of y in the second equation $2, 2(\frac{7+3y}{4}) + 5y = 23$

Algebra gives : $\frac{7+3+10y}{2} = 23$ → 7+13*y* = 46 → *y* = 3 and *x* = 4

2. Elimination Method: We multiply both equations with a constant such that when equation 1 is subtracted from equation 2, one variable is removed. Thus leaving a simple linear equation. Take the same example as above.

Multiply equation 1 with 1 → $4x - 3y = 7$
Multiply equation 2 with 2 → $4x + 10y = 46$
And substract 2 from 1 to find $0x - 13y = -39$

Therefore, $y = 3$ and use this result to substitute 3 for y in equation 1 or 2 to get $x = 4$.

Final note
There is one very important fact one should remember whenever solving equations: when you multiply two or more factors, their product can only be zero if at least one of their factors is zero.

For example: $x(x+3)(x-2) = 0$
Whenever an equation is set up this way, the first solution that has to cross your mind is that $x = 0$. It is clearly not the only one, and with the same reasoning you can find that $x = -3$ and $x = 2$ are the other solutions (both creating factors of 0).