## Chapter 9: Discounting Cash Flows

Real investments are cash outflows that create money in the future and are not financial instruments that trade in the financial markets, for example bonds. The present value is the market value of a portfolio that tracks the free cash that a project creates.

The cash flows of an asset are symbolized as numbers which belong to dates in time, with date 0 usually standing for the current date. The minus ( - ) or the plus $(+)$ tells us if it is a cash inflow or a cash outflow. Unlevered cash flows are cash flows that comes straight from the real assets of the project or organization.
EBIT stands for Earnings before interest and taxes. The raise or decrease in cash which is stated at the end of the cash flow statement, stands for the difference in the cash position of the organization for two following balance sheets. This is an accounting cash flow and it divides the cash flow into the sum of operating cash flows, investing cash flows, and financing cash flows.
Unlevered cash flow
= operating cash inflow

+ investing cash inflow (which is usually negative)
+ debt interest
- debt interest tax subsid

This is the case under the rules of GAAP, when following the rules of IFRS, the unlevered cash flow is computed by taking the net cash flow from operation activities and adding up the net cash flow from investing activities and finally reduce it with the debt interest subsidy.
Unlevered cash flow
= EBIT

+ depreciation and amortization
- change in working capital
- capital expenditures
+ sales of capital assets
- realized capital gains
+ realized capital losses
- EBIT X tax rate

Incremental cash flows are the cash flows that a organization gets when implementing the project. So, it is the difference between the cash flows with the project and the cash flows without the project in an organization. Projects should be appreciated by examining their incremental cash
flows to a organization.
A forecasting model consist out of 4 important issues:
1.Assumptions
2.Income statement
3.Cash flow statement
4.Balance sheet

## Assumptions

A very important assumption is the forecasting of sales. A lot of other features in the balance sheet and the income statement will depend on this. It is important, when you look for a trend in sales, that you examine the past performance, plans for products, the position of the organization in relation to its competitors and the trends of the industry, where the organization is in. A method to forecast the sales, is to base them on the expenses. The percentages are derived from past experiences.

## Income Statement

The cost of sales can be computed by relating the assumption of percentage of costs to the sales forecast. EBIT is now computed by taking the sales forecast and subtracting the cost of sales.
EBIT stands for the earnings before interest and taxes. Subtract interest from EBIT. This is then the earnings before taxes. Interest expense is a function of the expected future average interest rate and the expected future debt outstanding. Subtract taxes from earnings before taxes by applying the tax rate that you are forecasting for the year in question times the earnings before taxes.

The Cash flow statement has three features: operating cash flows, investment cash flows, and financing cash flows. In operating cash flows: net income, comes directly from the income statement, and then add back depreciation to the cash flow. The company's investment in working capital is then subtracted because it costs the organization money to raise its working capital.
Subtracted are also: raises in inventory, accounts receivable, and other current assets. Money spent on plant and equipment, or buying other companies, is an investment cash outflow.
$P P E=$ property, plant, equipment:
New PPE (net) $=$ Old PPE + capital investments - depreciation - sales of assets
The capital investments are computed by the New PPE minus the Old PPE plus the depreciation.
Balance sheet assets:
To predict PPE, there are several choices:

- direct method: if you can anticipate on capital expenditures and asset sales directly, then you can forecast PPE using the accounting relationships. Add the forecast expenditures to the old PPE and subtract your estimate of the depreciation expense and equipment sales.
- more of the same method: if the organization has substantial excess capacity and is not planning on adding capacity, then (1) simplifies: new PPE = old PPE less depreciation percentage of sales.


## Balance sheet liabilities

Long- and short-term debt $=$ the old debt numbers plus the new debt that the organization will issue. Current liabilities are directly related to sales, using a percentage. Deferred taxes are forecasted as a function of the forecast of PPE.

A single period return: $r=\frac{P_{1}-P_{0}}{P_{0}}$ where:
$P_{1}=$ date 1 investment value (plus any cash distributed) like dividends or coupons
$P_{0}=$ date 0 investment value
An investment of $€ 1$,- that grows to $€ 1,08$ has a rate of return of .08 (or 8 percent) over the period.
When $r$ is the interest rate (or rate of return per period) and all interest (profit) is reinvested, an investment of $P_{0}$ dollars at date 0 has a future value at date $t$ of: $P_{t}=P_{0}(1+r)^{t}$

The present value or discounted value is: $P_{0}=\frac{P_{t}}{(1+r)^{t}}$

Compound interest rates replicates the interest that is earned on interest and arises whenever interest earnings are reinvested in the account to enlarge the principal balance on which the investor earns future interest.

With a date 0 price of $P_{0}$ and a promise to pay a face value of $€ 100$,- at date $T$ and nothing prior to date $T$, the yield on the bond is the number $r$ that makes:
$r=\left(\frac{100}{P_{0}}\right)^{\frac{1}{T}}-1$

From now, present values are denoted as PV rather than $P_{0}$.

The present value of many cash flows merged is the sum of their individual present values:

$$
P V=\frac{C_{1}}{(1+r)^{1}}+\frac{C_{2}}{(1+r)^{2}}+\ldots+\frac{C_{T}}{(1+r)^{T}} \quad \text { the discount rate is } r .
$$

Nominal discount rates are rates acquired from observed rates of appreciation directly, which relate to nominal cash flows, the observed cash flows. These grow with inflation.

Real cash flows are adjustion for inflation, which take out the component of growth due to inflation. They need to be discounted at real discount rates, which are the nominal discount rates adjusted to inflation.

If $i$ is the rate of inflation per period, and $r$ is the appropriate nominal discount rate, the real discount rate per period is:
$r_{\text {real }}=\frac{1+r_{\text {no min } a l}}{1+i}-1$
Discounting nominal cash flows at nominal discount rates or inflation-adjusted cash flows at the appropriately computed real interest rates generates the same present value.

The algebraic representation of the infinite sum that is the present value of a perpetuity is:
$P V=\frac{C}{(1+r)}+\frac{C}{(1+r)^{2}}+\frac{C}{(1+r)^{3}}+\ldots$
Which simplifies to: $P V=\frac{C}{r}$
If $r$ is the discount rate per period, the present value of an annuity with payments commencing at date 1 and ending at date $T$ is
$P V=\frac{C}{r}\left(1-\frac{1}{(1+r)^{T}}\right)$

A growing perpetuity is a continuous cash flow stream that grows at a constant rate (denoted here as $g$ ) over time, for ever.

The value of a growing perpetuity with initial payment of C dollars one period from now is:

$$
P V=\frac{C}{r-g} \text { if } g \leq r
$$

A growing annuity is equal to a growing perpetuity apart from that the cash flows end at date $T$.
The present value of a $T$-period growing annuity, with an initial cash flow of C , commencing one period from now and with a growth rate of $g$, is given by:

$$
P V=\frac{c}{r-g}\left(1-\frac{(1+g)^{T}}{(1+r)^{T}}\right)
$$

Interest rates on all investments have a tendency to be quoted on an annualized basis. This annualization adjustment is imperfect for making comparisons among investments since it does not reflect the interest earned on reinvested interest.

To make use of the formulas developed above, where $r$ is the interest earned over a single period per dollar invested at the beginning of the period, one has to transform the rates that are quoted for financial securities back into rates per period and properly calculate the number of periods over which the future value or PV is taken.

Given the investment with the same interest rate $r$, but dissimilar compounding frequencies, the more common the compounding, the faster the growth rate of the investment. If the compounding frequency is $m$ times a year and the annualized rate is $r$, the amount accumulated from an investment of PV after $t$ years is:

$$
P_{t}=P V x\left(1+\frac{r}{m}\right)^{m t}
$$

In case of continuous compounding: $P_{t}=P V x e^{r t}$
Where $e$ is the base of the natural logarithm, approximately 2,718 .

