Chapter 23: Linear, second-order differential equations

A linear, second-order differential equation has the form:

$\ddot{y}(t) + \alpha_1\dot{y}(t) + \alpha_2 y(t) = \beta$

Solving linear, autonomous, second-order differential equations

1. Write in general form: $\ddot{y}(t) + \alpha_1\dot{y}(t) + \alpha_2 y(t) = \beta$

2. Use the **characteristic equation** $r^2 + \alpha_1 r + \alpha_2 = 0$ to find the roots (r). The lowest root is always the first root(r_1) you use in the homogeneous solution.

3. Compute the homogeneous solution:

   $y_h(t) =$ $C_1e^{r1t} + C_2e^{r2t}$ if r_1 is not equal to r_2

   $C_1e^{rt} + C_2te^{rt}$ if $r_1 = r_2 = r$

4. Compute particular solution:

   $y_p(t) = \beta / \alpha_2$ if α_2 is not 0

5. Report general solution

   $y(t) = y_h(t) + y_p(t)$

6. Compute constants C_1 and C_2 such that the initial conditions are satisfied

7. Report solution. Substitute the answers of step 6 into the general solution to find the end solution.

The solution of a linear, second-order differential equation only **converges** toward to steady-state equilibrium if the roots of the characteristic equation are negative.

Solving linear, non-autonomous, second-order differential equations

1. Write in general form: $\ddot{y}(t) + \alpha_1\dot{y}(t) + \alpha_2 y(t) = \beta\,(t)$

2. Use the **characteristic equation** $r^2 + \alpha_1 r + \alpha_2 = 0$ to find the roots (r). The lowest root is always the first root(r_1) you use in the homogeneous solution.

3. Compute the homogeneous solution:

   $y_h(t) =$ $C_1e^{r1t} + C_2e^{r2t}$ if r_1 is not equal to r_2

   $C_1e^{rt} + C_2te^{rt}$ if $r_1 = r_2 = r$

4. Compute the particular solution with the **method of undetermined coefficients:**

   $y_p(t) = A_nt^n + A_{n-1}t^{n-1} + \ldots + A_1t + A_0$

   Differentiate this formula to $\dot{y}(t)$ and $\ddot{y}(t)$. Plug these two formulas into the general form en than solve for all A's

5. Report general solution

   $y(t) = y_h(t) + y_p(t)$

6. Compute C_1 and C_2 such that the initial conditions are satisfied

7. Report solution. Substitute the answers of step 6 into the general solution to find the end solution.