Chapter 22: Nonlinear, first-order differential equations

For an autonomous, nonlinear, first-order differential equation the **initial-value problem** is expressed as:

$\dot{y} = g(y)$

$y(t_0) = y_0$

If g and its derivative are continuous and contain the point (t_0, y_0), then somewhere around t_0 there is a unique solution $y = \xi(t)$ that satisfies the two equations mentioned above.

We know that there is a solution in this case. However it can be very difficult to find this solution. A common method used in this case is a **qualitative analysis** often with a **phase diagram.** This method is used for nonlinear, first-order differential equations.

Qualitative analysis with phase diagram

$\dot{y}$ as a function of y

1. Draw the axes for the phase diagram with $\dot{y}$ on the vertical axis and y on the horizontal axis

2. Compute the steady-state solutions by solving $\dot{y}=0$ for y. Mark these points on the horizontal axis.

3. Compute the extremums by solving $d\dot{y}/dy=0$

   - When $d(d\dot{y}/dy) / dy > 0$, there is a minimum (convex)

   - When $d(d\dot{y}/dy) / dy < 0$, there is a maximum (concave)

4. Draw $\dot{y}$ as a function of y

5. Compute $d\dot{y}/dy$ in the steady-states. These steady-states were found in step 2.

   - When $d\dot{y}/dy < 0$, means the equilibrium point is a **stable point**

   - When $d\dot{y}/dy > 0$ means the equilibrium point is an **unstable point**