Chapter 21: Linear, first-order differential equations

Solving linear, autonomous, first-order differential equations:

1. Write in general form $\dot{y}(t) + \alpha\, y(t) = \beta$

2. Compute the **homogeneous solution:**

   $y_h(t) = Ce^{-\alpha t}$ if α is not equal to 0

3. Compute the **particular solution:**

   $y_p(t) = \beta/\alpha$ if α is not equal to 0

4. Report **general solution:**

   $y(t) = y_h(t) + y_p(t) = Ce^{-\alpha t} + \beta/\alpha$

5. Compute constants such that initial conditions are satisfied. Example of a condition:

   $y(t_0)=4$. Insert this into the general solution to compute the constants.

6. Report solution

The solution to a linear, autonomous, first-order differential equation converges to the steady-state equilibrium, no matter what the initial value is, only if $\alpha > 0$.

The **steady-state value** of a differential equation is the value of y at which y is stationary. It can be found by implying the condition $\dot{y} = 0$.

Solving linear, non-autonomous, first-order differential equations:

1. Write in general form $\dot{y}(t) + a(t)\, y(t) = b(t)$

2. Compute the **integrating factor** $e^{A(t)}$, with;

$$A(t) = \int a(t)dt$$

3. Compute the **integral**:

$$\int_{t_0}^{t} e^{A(s)}\, b(s)ds$$

4. Apply **theorem 21.5** to find the general solution:

$$Y(t) = e^{-A(t)}\left[\int_{t_0}^{t} e^{A(s)} b(s)ds + y(t_0)e^{A(t_0)}\right]$$

5. Plug in the initial conditions and report the solution