

Chapter 17: Differential equations

A difference equation is an equation for the change in a variable. A **differential equation** is similar to a difference equation and expresses how a variable changes over time. The difference is that with a differential equation time is considered to be a continuous variable.

An example of a time derivative of a variable is:

$$\dot{y} = dy/dt$$

We will study only **ordinary differential equations**: these equations contain only ordinary derivatives and no partial derivatives.

Differential equations can be classified according to:

1. Order

- **First-order** difference equations contain only the first derivative

$$\dot{y} + y = \alpha$$

- **Second-order** difference equations contains the first and second derivative

$$\ddot{y} + \dot{y} + y = \alpha$$

2. Autonomous/non-autonomous

- An equation is **autonomous** if it does not depend on time explicitly.

$$\dot{y}(t) = \alpha y(t) + \beta$$

- A **non-autonomous** equation depends explicitly on the time, t .

$$\dot{y}(t) = \alpha(t) y(t) + \beta(t) \quad (\text{Not only } y \text{ but also } \alpha \text{ and } \beta \text{ can depend on } t)$$

3. Linear/nonlinear

- A function is **linear** if all the y terms are raised to no power other than 1.

- If the y terms are raised to a power other than 1 then the equation is **nonlinear**.

4. Solution

A solution makes the differential equation true. A differential equation has usually many solutions.