Chapter 15: Concave programming

Concave programming is used for problems where the functions f and g are concave.

With concave programming and inequality constraints we do not use the normal Lagrangian but we use the **Kuhn-Tucker Lagrangian:**

$L(x, \lambda, \mu) = f(x) + \lambda g(x) + \mu x$

Slater's condition: if f and g are concave and differentiable, and there is a point (x^0_1, x^0_2) such that $g(x^0_1, x^0_2) > 0$, then there is a Lagrange multiplier λ^* such that the K-T conditions are necessary for the point to be a solution to the problem.

Steps for constrained optimization with inequality:

Format of the problem is: max y = … s.t. $x_1 \leq b$ etc.

1. Identify the objective function f(x)

2. Rewrite the constraint as $g(x) \geq 0$

3. Construct the Kuhn-Tucker Lagrangian:

   $L(x, \lambda, \mu) = f(x) + \lambda g(x) + \mu x$

4. Compute the FOCs

   - $dL/dx_1 = 0$, $dL/dx_2 = 0$ etc.

   - $\lambda * dL/d\lambda = 0$

   - $\mu_1 * dL/d\mu_1 = 0$, $\mu_2 * dL/d\mu_2 = 0$ etc.

5. Solve FOCs. Start with implying that $\lambda > 0$ and $x > 0$, leading to $\mu=0$