Lecture 6

Chapter 13,14

We looked at the short- and medium-run, now we study what determines the increase of the natural level of output in the long run. We will talk about economic growth and answer questions like why some countries are rich, while others are/remain poor and if countries converge to the same level of welfare eventually.
The real GDP growth differs around the world. Economic growth is defined as the increase in aggregate output over time. We are interested in income per capita (GDP/population) as a measure of the standard of living. It is striking that poor countries grow really fast. Would there be convergence around the globe? There are patterns of growth we can distinguish:

- Advanced economies
  The trend is negative.
- Developing economies
  The trend is positive
- Emerging economies
  The trend is positive and the economy is growing really fast.

Income per capita can be interpreted as a measure of happiness. Is a person happier when he becomes richer? There is indeed a positive relationship between wealth and life satisfaction. A high level of GDP per capita comes along with a high mean of life satisfaction (see slide 7). So the answer is yes, up to an annual income of $75,000.

To compare countries, we use the purchasing-power-parity (PPP) to determine exchange rate fluctuations and cross-country differences in price levels. We can see if the poor countries catch up with the rich ones, do they converge? With the PPP we can compare income per capita of rich and poor countries. A low level of income per capita belongs to a poor country and vice versa. In the graphs on slide 10 and 11 we can see that there is no convergence around the world, although it seems that the OECD countries do converge. They converge because these countries are similar.

The Solow growth model
With the Solow growth model we can understand where economic growth comes from. It will show that capital accumulation (through saving) can cause growth of income per worker, but cannot sustain it and that sustained growth of income per worker is the consequence of technological progress.
The relation between output and inputs is given by the **aggregate production function**:

$$Y = F(K, N, A)$$

Y = output
K = capital
N = labour
A = state of technology
F = how much is produced given the amounts of K, N and A

The level of technology determines how much can be produced with a certain amount of capital and labour at any time. Therefore, technology influences the production function. In chapter 14, we assume that A is fixed, but we will drop this assumption in chapter 15.

Constant returns to scale is a property of the economy in which, if the scale of operation is doubled – that is, if the quantities of capital K and labour N are doubled – then output Y will also double, so we get $xY = F(xK, xN)$. We can rewrite the aggregate production function as:
$$\frac{Y}{N} = \left(\frac{K}{N}, \frac{N}{N}\right) = F\left(\frac{K}{N}, 1\right)$$

As capital per worker K/N increases, so does output by worker Y/N. Capital accumulation (turning saving into productive assets) increases output per worker.
Be aware that this only holds when we talk about constant returns to scale!

As we can see in the graph on slide 19, increases in capital per worker lead to increases in output per worker. However, increases in capital per worker lead to smaller and smaller increases in output per worker. So, decreasing returns to scale. In mathematical terms we can say that the first-order derivative is positive, and the second-order derivative is negative.
Improvements in technology lead to a higher level of output per worker for a given level of capital per worker. The curve will shift upwards.
Capital accumulation by itself cannot sustain growth, it require sustained technological progress. The economy's rate of growth of output per worker is eventually determined by the economy's rate of technological progress.

The role of saving
The effects of the saving rate – the ratio of saving to GDP – on capital and output per worker are important because capital accumulation requires saving to turn into productive investment. An increase in the saving rate leads to a temporary higher growth of output per worker, and to a permanently higher level of output per worker. Assumptions to determine output in the long run:

1. The amount of capital determines the amount of output being produced
2. The amount of output determines the amount of saving and, in turn, the amount of capital accumulated over time

Back to the Solow growth model
To focus on the effect of capital accumulation on output, we make two simplifying assumptions:

- Population size, the participation rate and the unemployment rate are constant. So the employment rate N is constant.
- No technological progress, so A is constant.

Using these assumptions we can write the following equation $\frac{Yt}{N} = f(\frac{Kt}{N})$, with decreasing returns to capital per worker.

To see how output and investment are related, we make the two following assumptions:

- The economy is closed I = S + (T – G)
- Public saving (T – G) is equal to zero, so I = S
- Private saving is proportional to income, S = sY

Now we can say that It = sYt, in worker per terms this is $\frac{It}{N} = s\frac{Yt}{N}$. This is a concave curve below the output curve.

The evolution of capital stock is given by $K(t+1) = (1-\delta)Kt + It$, where δ is the rate of depreciation. Using the assumptions that A and N are constant, we can rewrite the equation:

$$\frac{K(t+1)}{N} = (1-\delta)\frac{Kt}{N} + s\frac{Yt}{N}$$

$$\frac{K(t+1)}{N} - \frac{Kt}{N} = s\frac{Yt}{N} - \delta\frac{Kt}{N}$$

In words, the change in the capital stock per worker is equal to investment per worker (saving per worker) minus depreciation per worker (required investment).

The change in capital per worker from year t to year t + 1 is represented by $\frac{K(t+1)}{N} - \frac{Kt}{N}$.
The investment per worker during year t is represented by $sf(\frac{Kt}{N})$.
The depreciation per worker during year t is represented by $\delta\frac{Kt}{N}$.

Over time the capital stock per worker increases until we are in the steady state where investment per worker (saving per worker) equals depreciation per worker (required investment per worker).
The stock of capital rises when sf > δ. In the long run δ = sf.
If the savings rate increases, sf will shift upwards.

If investment per worker exceeds depreciation per worker, the change in capital per worker is positive: capital per worker increases.
If investment per worker is less than depreciation per worker, the change in capital per worker is negative: capital per worker decreases.

Since in the steady state the level of capital per worker remains unchanged, that is

$$\frac{K(t+1)}{N} - \frac{Kt}{N} = sf\left(\frac{Kt}{N}\right) - \delta\frac{Kt}{N} = 0$$

We find

$$sf\left(\frac{K*}{N}\right) = \delta\frac{K*}{N}$$

So output per worker in the steady state is

$$\frac{Y*}{N} = f\left(\frac{K*}{N}\right)$$

C/N in the steady state

$$Y = C + I = C + sY \ gives \ \frac{Yt}{N} = \frac{C}{N} + s\frac{Yt}{N}$$

There are three observations on saving:

1. The saving rate has no effect on the long-run growth rate of output per worker
2. Nonetheless, the saving rate determines the level of output per worker in the long run. Other things equal, countries with a higher saving rate will achieve higher output per worker in the long run
3. An increase in the saving rate will lead to higher growth of output per worker for some time, but not forever

If the savings rate increases, there will be an increase in output per worker too so the level of output goes up. This leads to temporary growth. There will be a movement from the old steady state to the new one.

Saving and consumption

The higher the savings rate, the higher you move along the production function so output will increase. But consumption will not always increase. So, An increase in the saving rate always leads to an increase in the level of output per worker, but not in consumption per worker. The savings rate fluctuates between 0 and 1. If the savings rate is 0, capital is also 0. investment is 0 so output per worker will become also 0. C/N will be 0, so consumption is 0.
So if s = 0, C/N = 0.
If the savings rate is 1, the level of capital per worker in the steady state will be very high. Everything you produce is saved so all income is saved. The consumption thus will be 0.
So if s = 1, C/N = 0.

Optimal saving rate

There is an optimal saving rate $0 < s_G < 1$ that maximizes steady-state consumption per worker. This is called the **golden rule steady state**. In the steady state, consumption per worker equals

$$\frac{C*}{N} = \frac{Y*}{N} - sf\left(\frac{K*}{N}\right) = \frac{Y*}{N} - \delta\frac{K*}{N} = f\left(\frac{K*}{N}\right) - \delta\frac{K*}{N}$$

To find the optimal saving rate you take the first derivative to the capital per worker function. This will be the slope and set it equal to 0.

$$\frac{\partial(C*/N)}{\partial(K*/N)} = \frac{\partial f(K*/N)}{\partial(K*/N)} - \delta = 0$$

This means: marginal product of capital = depreciation rate. Slope $f\left(\frac{K}{N}\right) = \delta$.
The marginal product of capital = the depreciation rate.

On slide 41 an 42 you can graphically see the golden rule. Consumption equals the difference between the red and the blue line. There is a maximum where the slope of Y/N equals the slope of δ.

Extensions of the Solow model

The Solow model so far is simplified where employment is constant and where there is no technological progress. Some extensions to the model are:

- Human capital
  Some workers are more productive than others because they learn: education, on-the-job-training).
- Populations grow
- Technology improves

An economy with many highly skilled workers is likely to be much more productive than an economy in which most workers cannot read or write. The set of skills of the workers in the economy is called human capital (H), and is included in the extended model:

$$\frac{Y}{N} = f(\frac{K}{N}, \frac{H}{N})$$

An increase in capital per worker or the average skill of workers (through education and on-the-job-training) leads to an increase in output per worker.

Models that generate steady growth even without technological progress are called models of endogenous growth, where growth depends on variables such as the saving rate and the rate of spending on education.

Output per worker depends on the level of both physical capital per worker and human capital per worker. The questions asked in this lecture can be answered:

- Technological progress is related to the level of human capital in the economy
- A better educated labour force lead to a higher rate of technological progress
- Incorporating technological progress makes economic growth sustainable