Chapter 5: Interest Rates

Interest rate quotes

Interest rates are set by market forces, especially the supply and demand of funds. When the savings are high (high supply) and the borrowing is low (low demand), the interest rates will be low. Besides market forces, interest rates are also influenced by expected inflation and risk. Interest rates are often quoted for different time intervals, such as monthly or annual, and therefore it is necessary to adjust the interest rate to a time period that matches the cash flows.

The *effective annual rate (EAR)* or the *annual percentage yield (APY)* indicates the total amount of interest that will be earned at the end of one year. The EAR can be used as a discount rate for annual cash flows.

We can convert a discount rate for one period to an equivalent discount rate for n periods. The discount rate for an n-year time interval, given an EAR r, can be calculated with this formula:

Equivalent n-period discount rate = $(1 + r)^n - 1$

When you compute a rate over more than one period, n is larger than 1. n is smaller than 1 if you compute a rate over a fraction of a period.

The *annual percentage rate (APR)* indicates the amount of *simple interest*; which is the interest earned in one year without the effect of compounding. APR this is the most common way to note interest rates. The APR quote is less than the actual amount of interest you'll earn, because APR doesn't include the effect of compounding. For this reason, the APR itself cannot be used as a discount rate.

Interest rate per compounding period = APR / number of compounding periods per year (m).

We can convert an APR to an EAR with this formula:

$$1 + EAR = (1 + \frac{APR}{m})^{m}$$

This formula gives the effective annual rate corresponding to an APR. The EAR increases with the compounding frequency, for a given APR. For example, given an 6% APR and a annual compounding interval, the effective annual rate is 6% . $[(1 + \frac{0.06}{1})^{1} - 1]$Given the same APR and a monthly compounding interval, the effective annual rate is 6.1678% . $[(1 + \frac{0.06}{12})^{12} - 1]$

Before you're able to evaluate the PV or FV of set of cash flows, you have to convert the APR to a discount rate per compounding interval (use: $(1 + r)^n - 1$) or coCnvert the APR to an EAR (use: $1 + EAR = (1 + \frac{APR}{m})^{m}$)

Loans

Many loans, such as mortgages and car loans, are amortizing loans. An *amortizing loan* is a loan on which the borrower makes monthly payments that include interest on the loan plus some part of the loan balance. Amortizing loans are quoted in terms of an APR with monthly compounding.

The amount you pay for your loan every month includes interest and repayment of part of the principal, reducing the amount you still owe (the loan balance).

The loan balance decreases each month. For that reason, the accruing interest on that loan balance will decrease too. Because the loan payment is equal every month, the repayment component will increase. So, the outstanding balance on an amortizing loan differs every month. You can calculate the outstanding loan balance by determining the PV of the remaining loan payments, using the loan rate as the discount rate.

Determinants of interest rates

We will discuss some determinants of interest rates, such as inflation, current economic activity, and expectation of future growth.

Inflation affects the evaluating of interest rates being quoted by banks and other financial institutions. Those quoted interest rates are *nominal interest rates*: interest rates that indicate the rate at which money will grow if invested in a certain period of time. The *real interest* rate indicated the rate of growth of purchasing power after adjusting for inflation. You can compute the *rate of growth of purchasing power* with this formula:
Growth in purchasing power = 1 + real rate
= (1 + nominal rate) / (1+inflation rate)
= growth of money / growth of prices

The real interest rate:
Real rate = (nominal rate – inflation rate) / (1+ inflation rate)
≈ nominal rate – inflation rate

Nominal interest rate tends to move with inflation. The nominal interest rate is high when inflation is high and vice versa. Savings depends on the growth in purchasing power individuals can expect, this growth in purchasing power is given by the real interest rate. So, when the inflation rate is high, the country needs a higher nominal interest rate to induce individuals to save.

Interest rates also affect the incentive of firm's to raise capital and invest. When the interest rate rises, the PV of the benefits will fall and companies will not invest because the investment isn't profitable. When the interest rates are high, you're discounting the positive cash flows at a higher rate, which reduces the PV.

The Federal Reserve in the USA and central banks in other countries try to guide the economy by influencing interest rates. If the economy is slowing, they will lower the interest rates in attempt to stimulate investment in the economy. They attempt to reduce investment in the economy by raising the interest rates.

Interest rates depend on the horizon of the investment or the loan according to the term structure of interest rates. The term structure of interest is the relationship between the investment term and the interest rate. The *yield curve* is a plot of bond yields as a function of the bonds' maturity date, with on the x-axis the term and on the y-axis the interest rate. *A risk-free interest rate* is the interest rate at which money can be borrowed or lent without risk over a given period.

To compute the PV and the FV of a risk-free cash flow over different investment horizons, we can use the term structure. You should discount cash flows using the discount rate that is appropriate for their horizon. So, you should discount a cash flow received in two years at the two-year interest rate and a cash flow received in eight years at the eight-year interest rate. You need to match the term of the cash flow with the term of the interest rate.

The present value of a cash flow stream using a term structure of discount rates:

$$PV = \frac{C1}{(1+r1)} + \frac{C2}{(1+r2)^2} + \ \dots\dots + \frac{CN}{(1+rN)^N}$$

Note that you use a different discount rate for each cash flow. This discount rate is based on the rate from the yield curve with the same term. So, you cannot use the formulas discussed before (e.g. annuity and perpetuity) when discount rates vary with the horizon.

The *federal funds rate* is the overnight loan rate charged by banks with excess reserves at a Federal Reserve bank (called federal funds) to banks that need additional funds to meet reserve requirements.

A yield curve changes over time. The shape of the yield curve depends to a larger extent on the expectations of investors of future economic growth and interest rates, because these expectations have a major effect on the willingness to lend or borrow for longer terms. To attract investors, long-term interest will tend to be higher than short-term interest rates, if interest rates are expected to rise. Then the yield curve will be steep. Otherwise, if interest rates are expected to fall, short-term interest rates will tend to be higher than long-term interest rates to attract borrowers. In this case, the yield curve will be descending, this is a negative forecast for economic growth.

Opportunity cost of capital

To evaluate cash flows we will base the discount rate that we use on the opportunity cost of capital of the investor. The opportunity cost of capital is the best available expected return offered in the market on an investment of comparable risk and term to the cash flow being discounted. It is the return the investor forgoes on an alternative investment of equivalent risk and term when the investor takes on a new investment.