Chapter 4: Valuing cash flow streams

Valuing a stream of cash flows

Almost all investment opportunities have multiple cash flows that occur at different points in time. Using the rules of cash flow valuation, we compute the present value of a stream of cash flows in two steps.

1. Compute the present value of each individual cash flow.
2. When the cash flows are in common units of dollars today, we can combine them.

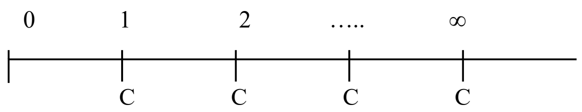
This leads to the following formula for the present value of a stream of cash flows:

$$PV = C0 + \frac{C1}{(1+r)} + \frac{C2}{(1+r)^2} + \dots\dots + \frac{CN}{(1+r)^N}$$

We will consider two types of cash flow streams; perpetuities and annuities.

Perpetuities

A p*erpetuity* is a stream of equal cash flows that occurs at regular intervals and lasts forever. One example is the consol, a British government bond. The *consol* promises the owner a fixed cash flow every year, forever.
The first cash flows of a perpetuity arrives at the end of the first period

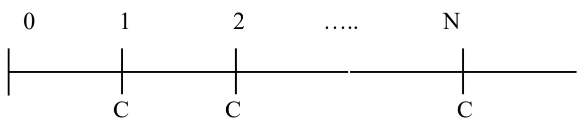


The present value of a perpetuity:

$$PV = \frac{C}{(1+r)} + \frac{C}{(1+r)^2} + \dots\dots = \frac{\boldsymbol{C}}{\boldsymbol{r}}$$

Annuities

An annuity is a stream of equal cash flows arriving at a regular interval and ending after a specified time period. A perpetuity is infinite, while an annuity ends after some fixed number of payments.



The present value of an annuity**:**

$$PV = \frac{C}{(1+r)} + \frac{C}{(1+r)^2} + \ldots\ldots + \frac{C}{(1+r)^N}$$

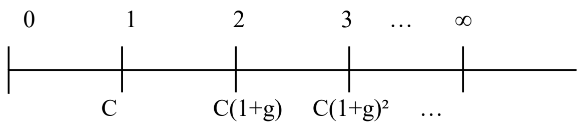
$$PV = \frac{C}{r}\left[1 - \frac{1}{(1+r)^N}\right]$$

The future value of an annuity:

$$FV = \frac{C}{r}\left[(1+r)^N - 1\right]$$

Growing cash flows

A *growing perpetuity* is a stream of cash flows that occurs at regular intervals and grows at a constant rate forever.

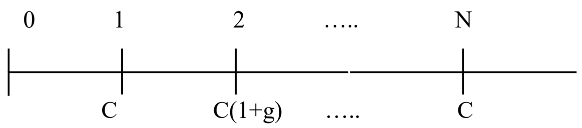


We follow the same logic to calculate the present value of a growing perpetuity as for a regular perpetuity. But now we also take the growth rate, g, in consideration.

The present value of a constant growing perpetuity:

$$PV = \frac{C}{r-g}$$

A *growing annuity* is an annuity with the cash flow growing at a fixed growth factor.



The present value of a constant growing annuity:

$$PV = \frac{C}{r-g}\left[1 - \left(\frac{1+g}{1+r}\right)^N\right]$$

Other variables

Sometimes we know the PV or FV, but we don't know one of the other variables (e.g. C, n or r). In such situations, we can use the PV or FV as inputs to solve for the variable we are interested in. Some examples:

1. We don't know the cash flow, but we know the PV.
   To solve this problem, you enter all the known variables (also the PV) into the formula and you solve the equation for C.

   In case of a loan payment (annuity) : The periodic payment on an N-period loan with principal P and interest rate r is:

$$C = \frac{P}{\frac{1}{r}[1 - \frac{1}{(1+r)^{N}}]}$$

2. We don't know the interest rate, but we know the PV and the cash flows.
   In a situation with an unknown interest rate, the interest rate is called the internal rate of return (IRR). The IRR is the interest rate at which the PV of the benefits exactly offsets the PV of the costs. Solving this problem is the same as the previous one: you enter all the known variables into the same formula as with the annuity formulas and solve the equation for r.

3. We don't know the number of periods "n", but we know the interest rate, present value and future value. It is most easy to calculate the years, n, with a formula in excel.